Spring 09 PreCal Evaluation

MTY Academy Spring 09-PreCal

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1. Find all solutions of each equation below. Express the solutions in radians. Show your work in detail!

   (1) $\sin x + \dfrac{\sqrt{3}}{2} = 0$

   (2) $\sqrt{3}\cot x + 1 = 0$

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2. Find all solutions to each of the following equations in the interval $[\,0, 2\pi)$.

   (1) $\csc x - 2 = -4$

   (2) $(3\tan x - \sqrt{3})(\sec x + 2) = 0$

   (3) $2\sin^2 x - 5\sin x = -2$

   (4) $2\sec^2 x + 3\sec x = 2$

   (5) $\sqrt{3}\sec^2 x - 2\tan x - 2\sqrt{3} = 0$

   (6) $2\cos^2 x - \sin x + 1 = 0$

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3. Find the exact value of each expression. Using indicated formula.

   (1) $\cos\left(-\dfrac{5\pi}{12}\right)$ (The sum-difference formula)

   (2) $\csc\dfrac{\pi}{12}$ (The sum-difference angles formula)

   (3) $\sec 15^\circ$ (The sum-difference angles formula)

   (4) $\sin 80^\circ \cos 20^\circ - \cos 80^\circ \sin 20^\circ$ (Any formula)

   (5) $\tan 195^\circ$ (The half-angle formula)

   (6) $\sin(-75^\circ)$ (The half-angle formula)

   (7) $\csc 195^\circ$ (Any formula)

   (8) $\cot 75^\circ$ (The half-angle formula)

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4. Prove that

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

5. Given that $\cos\alpha = -\frac{1}{3}$ with α in quadrant II; and $\sin\beta = -\frac{3}{5}$ with β in quadrant IV, find the exact value of each of the following expressions.

   (1) Draw the angles α and β in standard position.

   (2) $\sin(\alpha+\beta)$

   (3) $\cot(\alpha+\beta)$

   (4) $\sec(\alpha-\beta)$

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6. Simplify the following expression to a trigonometric function of a single angle and then evaluate.

$$\frac{\tan 70^\circ + \tan 35^\circ}{1 - \tan 70^\circ \tan 35^\circ}$$

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7. Assume that $\csc\theta = -4$, and $\cos\theta < 0$.

   (1) Draw a right triangle for the angle θ.

   (2) Find the exact value of $\sin 2\theta$

   (3) Find the exact value of $\cos 2\theta$

   (4) Find the exact value of $\cos 4\theta$

   (5) Find the exact value of $\cot 4\theta$

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8. Show that $\cot 2\theta = \dfrac{1}{2}(\cot\theta - \tan\theta)$

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9. Prove the following identity.

$$\frac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta} = \cos 2\theta$$

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10. Given that $\tan\theta = -\frac{2}{3}$, $\frac{\pi}{2} < \theta < \pi$, find the exact value of each of the following expressions.

    (1) $\sin\dfrac{\theta}{2}$

    (2) $\cos\dfrac{\theta}{2}$

    (3) $\cot\dfrac{\theta}{2}$

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11. Find exact value of $\cos 37.5^\circ \sin 7.5^\circ$

12. Use sum-to-product formulas to rewrite each expression as a product. Simplify where possible.

    (1) $\cos 3x + \cos 6x$

    (2) $\cos\theta - \sin 3\theta$

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13. Rewrite each of the following as a single sine.

    (1) $4\sin x - 7\cos x$

    (2) $3\cos x + 5\sin x$

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14. Rewrite each of the following as a single cosine.

    (1) $4\sin x - 7\cos x$

    (2) $3\cos x + 2\sin x$

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15. Find all solutions of the following equations in $[0, 2\pi)$.

    (1) $\sin x + \sin(3x) = 0$

    (2) $\cos x - \sin x = -1$

    (3) $4\sin x + 3\cos x = 3$
    (round up to the nearest degree)

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16. Solve each triangle (Find all missing angles and sides), using the law of sines.

    (1) $\beta = 70°, \quad \gamma = 10°, \quad b = 5$

    (2) $a = 6, \quad b = 8, \quad \alpha = 35°$

    (3) $a = 3, \quad b = 6, \quad \alpha = 45°$

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17. Find the area of each of the following triangles. Rounded up to the nearest hundredth.

    (1) $a = 2,\ c = 1,\ \beta = 10°$

    (2) $a = 6,\ b = 4,\ \gamma = 70°$

    (3) $a = 4,\ b = 8,\ c = 6$

    (4) $a = 2,\ \alpha = 40°,\ \beta = 20°$