# Fall 09 PreCal Evaluation



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1. For each given angle below, draw the angle in standard position, mark the reference angle in your sketch, and find the measure of the reference angle.

   (1) $-430^\circ$

   (2) $\frac{12\pi}{5}$

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2. Let $(-4, -2)$ be a point on the terminal side of an angle $\theta$ in the standard position.

   (1) Draw the angle $\theta$ in a coordinate system. Indicate the reference angle of $\theta$. Also draw a corresponding right triangle and indicate the lengths of sides on the triangle.

   (2) Find exact values of the six trigonometric functions of $\theta$. Simplify your answers.

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3. Suppose that the terminal side of an angle $\theta$ in the standard position is in quadrant II and $\tan\theta = -\frac{3}{7}$.

   (1) Draw the angle $\theta$ in a coordinate system. Indicate the reference angle of $\theta$. Also draw a corresponding right triangle and indicate the lengths of sides on the triangle.

   (2) Find exact values of the six trigonometric functions of $\theta$. Simplify your answers.

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4. Find each of the following values, using the unit circle, or right triangles in coordinate system. Simplify your answers (no radicals are allowed in the denominator). **Do not use a calculator.**

   (1) $\sec\dfrac{\pi}{6} + 3\csc\dfrac{\pi}{4}$

   (2) $\left(\tan\dfrac{\pi}{3}\right)^2 + \left(\cos\dfrac{\pi}{6}\right)^2$

   (3) $\dfrac{\cos 120^\circ}{\cot(-150^\circ) - \csc(-240^\circ)}$

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5. Given $f(x) = \dfrac{4x-5}{x-2}$

   (1) find the inverse $f^{-1}(x)$ of $f(x)$.

   (2) find the range of $f(x)$.

   (3) find the range of $f^{-1}(x)$.

6. Prove that $\csc(90^\circ - \theta) = \sec\theta$, where $\theta$ is a acute angle.

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7. Use the **definition** to determine whether each of the following functions is even, odd, or neither.

   (1) $f(x) = \dfrac{x}{x^4 - 9}$

   (2) $f(x) = |2 + \sin x|$

   (3) $f(x) = \sin x \cos x$

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8. For each of the following functions, find the amplitude and the period. Then sketch one complete cycle of each graph by finding high and low critical points and x-intercepts. Label these five points.

   (1) $y = 3 + \sin\left(x + \frac{\pi}{6}\right)$

   (2) $y = 4 - 5\cos\dfrac{1}{3}(x + \pi)$

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9. Draw the graph of each of the following functions with exactly **ONE** cycle. Label the turning point. Find the period and **ALL** asymptotes.

   (1) $y = -2 + 5\tan\frac{1}{2}\left(x + \frac{\pi}{3}\right)$

   (2) $y = 2 + \cot\frac{1}{2}(x - \frac{\pi}{2})$

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10. Draw the graph of each of the following functions with exactly **ONE** cycle. Find the period and ALL asymptotes. Also find the range.

    (1) $y = 2 - 4\csc\frac{1}{2}(x - \frac{\pi}{2})$

    (2) $y = 6 + 3\csc\frac{1}{2}(x + \frac{\pi}{4})$

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11. Use a calculator to find the value of $y$ in degree to two decimal places. Draw each angle in standard position.

    (1) $y = \csc^{-1}(-134)$

    (2) $y = \sec^{-1}(-345)$

    (3) $y = \cot^{-1}(-239)$

12. Find exact value of each of the following expressions by drawing its corresponding angle in standard position.

(1) $\tan\left[\csc^{-1}\left(-\frac{7}{3}\right)\right]$

(2) $\sec\left[\sin^{-1}\left(-\frac{3}{4}\right)\right]$

(3) $\cos\left[\tan^{-1}\left(-\frac{5}{2}\right)\right]$

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13. Find exact value of each of the following expressions.

(1) $\sin^{-1}\left(\sin\frac{4\pi}{3}\right)$

(2) $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

(3) $\csc^{-1}\left(\csc(-250^\circ)\right)$

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14. Prove each of the following identities. Show your work in detail.

(1) $\cos x\tan x - \csc x = -\cos x\cot x$

(2) $\tan x + \cot x = \csc x\sec x$

(3) $\tan x(\cos x + \cot x\cos x) = \sin x + \cos x$

(4) $\dfrac{\csc^2 x - 1}{\cos x} = \cot x\csc x$

(5) $\left(\tan x + \dfrac{1}{\cot x}\right)\left(\cot x + \dfrac{1}{\tan x}\right) = 4$

(6) $\dfrac{\sin^3 x - \cos^3 x}{\sin x + \cos x} = \dfrac{\csc^2 x - \cot x - 2\cos^2 x}{1 - \cot^2 x}$