## Spring 08 Algebra Evaluation

1. Find the domain of each of the following rational functions. Show your work in detail.
(1) $f(x)=\frac{x^{2}-4}{2 x^{3}-x^{2}-15 x}$
(2) $f(x)=\frac{x^{2}-4}{2 x^{2}+x+2}$
2. Let $f(x)=9-\frac{1}{(x+4)^{2}}$.
(1) Find the domain of $f(x)$.
(2) Find the intercepts.
(3) Sketch the graph of $f(x)$ by using transformations. Describe your transformations in words. Label all your graphs in order. Indicate any asymptotes and intercepts on your graph.
(4) Write any asymptotes as an equation.
(5) Find the range of $f(x)$.
3. Let $f(x)=\frac{x^{2}-4 x}{2 x^{2}-5 x-12}$
(1) Find the domain.
(2) Find any vertical asymptote(s) and horizontal asymptote.
(3) Find $y$-intercept and $x$-intercept(s).
4. Let $f(x)=\frac{x-1}{x^{2}-9}$.
(1) Find the domain of $f(x)$.
(2) Find all intercept.
(3) Find the asymptotes, if any.
(4) Sketch the graph of $f(x)$ by using the information obtained above, plus plugging at least two points.
5. Let $f(x)=\frac{\left(x^{2}-1\right)(x+2)}{x^{2}-2 x-3}$.
(1) Find the domain of $f(x)$.
(2) Find all intercept.
(3) Find any types of asymptotes, if any.
(4) Sketch the graph of $f(x)$ by using the information obtained above, plus plugging at least two points.
6. Solve each of the following inequalities, using the testing-point method (a sign-chart). Graph the solution set and write the solution set in interval notation.
(1) $\frac{-7}{x^{2}+4 x-12}<1$
(2) $\frac{x-4}{x+5} \leq \frac{1}{x-3}$
(3) $\frac{1}{x+5} \geq \frac{1-2 x}{x^{2}+2 x-15}$
(4) $-1 \leq \frac{x+4}{3-x} \leq 3$
7. Determine whether each of the following functions is one-to-one. Give reasons to support your answers.
(1) $f(x)=\frac{3}{|x|+1}$
(2) $f(x)=x^{3}-4 x$
8. Use the definition to prove each of the following functions is one-to-one.
(1) $f(x)=2-\sqrt{4-3 x}$
(2) $f(x)=x^{3}+2 x$
9. Let $f(x)=\frac{3-2 x}{x+5}$.
(1) Find the inverse function $f^{-1}$.
(2) Identify the domain and range for BOTH functions.
10. Let $f(x)=3^{x+4}-5$.
(1) Find all intercept;
(2) Find the horizontal asymptote;
(3) Sketch the graph of $f(x)$ using transformations. Indicate all information found above.
(4) Find the domain and the range.
11. Let $f(x)=-e^{x-2}-3$.
(1) Find the y-intercept;
(2) Find the x -intercept;
(3) Sketch the graph by transformations. Describe the transformations in words.
(4) Find the horizontal asymptote;
(5) Find the range.
12. Solve each of the following equations. Show your work in detail.
(1) $3^{x^{2}-4}=\left(\frac{1}{27}\right)^{x}$
(2) $5 \cdot 2^{x}-3 \cdot 2^{x}+2 \cdot 2^{x}=8$
(3) $3^{2 x-1}=7$
(4) $6=3^{2 x-1}$
(5) $\quad 4 e^{3 x}=3^{x+1}$
(6) $25^{x}-2 \cdot 5^{x}=8$
13. Find the value of $\log _{\frac{1}{2}} \sqrt[3]{16}=$ without using a calculator. Show your work.
14. Rewrite $\log _{b} \sqrt{\frac{x^{3} y^{4}}{b x y^{5}}}$ as a sum or difference of simpler logarithms.
15. Rewrite $3 \ln x-\ln y-4 \ln (2 x)+\frac{1}{3} \ln (y-1)$ as a single logarithm.
16. Solve each of the following equations. Verify your answers. Box the solutions.
(1) $\log _{x} 25=2$
(2) $\log _{2} x-\log _{2}(x-1)=1$
(3) $\log _{2}(x+1)+\log _{2}(x-2)=2$
(4) $4(\log x)^{2}+5 \log x=6$
(5) $4(\ln (2 x-1))^{2}+8 \ln (2 x-1)-5=0$
