

Spring 08 Algebra Evaluation

MTY Academy

Spring 08-Algebra

1. Find the domain of each of the following rational functions. Show your work in detail.

(1)
$$f(x) = \frac{x^2 - 4}{2x^3 - x^2 - 15x}$$

(2) $f(x) = \frac{x^2 - 4}{x^2 - 4}$

(2)
$$f(x) = \frac{1}{2x^2 + x + 2}$$

2. Let
$$f(x) = 9 - \frac{1}{(x+4)^2}$$
.

- (1) Find the domain of f(x).
- (2) Find the intercepts.
- (3) Sketch the graph of f(x) by using transformations. Describe your transformations in words. Label all your graphs in order. Indicate any asymptotes and intercepts on your graph.
- (4) Write any asymptotes as an equation.
- (5) Find the range of f(x).

3. Let
$$f(x) = \frac{x^2 - 4x}{2x^2 - 5x - 12}$$

- (1) Find the domain.
- (2) Find any vertical asymptote(s) and horizontal asymptote.
- (3) Find y-intercept and x-intercept(s).
- 4. Let $f(x) = \frac{x-1}{x^2-9}$.
 - (1) Find the domain of f(x).
 - (2) Find all intercept.
 - (3) Find the asymptotes, if any.
 - (4) Sketch the graph of f(x) by using the information obtained above, plus plugging at least two points.

5. Let
$$f(x) = \frac{(x^2 - 1)(x + 2)}{x^2 - 2x - 3}$$

(1) Find the domain of f(x).

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- (2) Find all intercept.
- (3) Find any types of asymptotes, if any.
- (4) Sketch the graph of f(x) by using the information obtained above, plus plugging at least two points.
- 6. Solve each of the following inequalities, using the testing-point method (a sign-chart). Graph the solution set and write the solution set in interval notation.

(1)
$$\frac{-7}{x^2 + 4x - 12} < 1$$

(2)
$$\frac{x-4}{x+5} \le \frac{1}{x-3}$$

(3)
$$\frac{1}{x+5} \ge \frac{1-2x}{x^2+2x-15}$$

(4)
$$-1 \le \frac{x+4}{3-x} \le 3$$

7. Determine whether each of the following functions is one-to-one. Give reasons to support your answers.

(1)
$$f(x) = \frac{3}{|x|+1}$$

(2)
$$f(x) = x^3 - 4x$$

- 8. Use the definition to prove each of the following functions is one-to-one.
 - (1) $f(x) = 2 \sqrt{4 3x}$
 - (2) $f(x) = x^3 + 2x$
- 9. Let $f(x) = \frac{3-2x}{x+5}$.
 - (1) Find the inverse function f^{-1} .
 - (2) Identify the domain and range for BOTH functions.
- 10. Let $f(x) = 3^{x+4} 5$.
 - (1) Find all intercept;
 - (2) Find the horizontal asymptote;
 - (3) Sketch the graph of f(x) using transformations. Indicate all information found above.
 - (4) Find the domain and the range.

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11. Let $f(x) = -e^{x-2} - 3$.

- (1) Find the y-intercept;
- (2) Find the x-intercept;
- (3) Sketch the graph by transformations. Describe the transformations in words.
- (4) Find the horizontal asymptote;
- (5) Find the range.
- 12. Solve each of the following equations. Show your work in detail.
 - (1) $3^{x^2-4} = (\frac{1}{27})^x$
 - (2) $5 \cdot 2^x 3 \cdot 2^x + 2 \cdot 2^x = 8$
 - (3) $3^{2x-1} = 7$
 - (4) $6 = 3^{2x-1}$
 - (5) $4e^{3x} = 3^{x+1}$
 - $(6) \qquad 25^x 2 \cdot 5^x = 8$
- 13. Find the value of $\log_{\frac{1}{2}} \sqrt[3]{16}$ = without using a calculator. Show your work.
- 14. Rewrite $\log_b \sqrt{\frac{x^3y^4}{bxy^5}}$ as a sum or difference of simpler logarithms.
- 15. Rewrite $3 \ln x \ln y 4 \ln(2x) + \frac{1}{3} \ln(y-1)$ as a single logarithm.
- 16. Solve each of the following equations. Verify your answers. Box the solutions.
 - (1) $\log_x 25 = 2$
 - (2) $\log_2 x \log_2(x-1) = 1$
 - (3) $\log_2(x+1) + \log_2(x-2) = 2$
 - (4) $4(\log x)^2 + 5\log x = 6$
 - (5) $4(\ln(2x-1))^2 + 8\ln(2x-1) 5 = 0$