Fall 08 Algebra Evaluation

1. Find each of the following and write your final answers in standard form $a + bi$.

   (1) $(13 + 2i) - 3(5 + 7i) - 2(5 - 4i)$

   (2) $(4 + 2i)(4 - 2i) - (2 + 7i)^2$

---

2. Rationalize the denominator for $\dfrac{2i}{2 - 7i}$ and write your final answers in standard form $a + bi$.

---

3. Use the discriminant $b^2 - 4ac$ to determine the number of solutions to $(x-3)(2x-5) = -4$,

---

4. For each complex number z below, find the length $|z|$ of z.

   (1) $z = (2 + 3i)(1 - 5i)$

   (2) $z = \dfrac{1 + 4i}{3 - 2i}$

---

5. Let $z = a + bi$ and $w = c + di$. Prove that $|zw| = |z|\ |w|$.

---

6. Assume that $f(x)$ is a linear function. If $f(2) = -8$ and $f(-3) = 2$, find $f(x)$.

---

7. Write the quadratic functions $f(x) = 2(x - 3)^2 - (2x - 8)$ in general form.

---

8. Write the quadratic function $f(x) = 3x^2 - 5x + 1$ in standard form.

---

9. Let $f(x) = -(x + 3)^2 + 4$.

   (1) Find the vertex of the parabola of $f(x)$;

   (2) Find the intercepts;

(3) Sketch the parabola of $f(x)$. Label the vertex, all intercepts, and symmetric axis.

(4) Find the minimum or maximum value of the function, if any.

(5) Find the range of $f(x)$.

---

10. Let $f(x) = x^2 - 2x + 6$.

(1) Find the vertex of the parabola of $f(x)$;

(2) Find the intercepts;

(3) Sketch the parabola of $f(x)$. Label the vertex, all intercepts, and symmetric axis.

---

11. Let $f(x) = -2(x+3)^2 - 1$.

(1) Find the vertex of the parabola of $f(x)$;

(2) Find the x- and y-intercepts;

(3) Sketch the parabola of $f(x)$ using transformations. Identify the starting function. Describe all transformations in words. Label your graphs in order. Label the vertex.

---

12. **Find** the values of a and b so that the graph of $f(x) = a(x-b)^2 + 2$ passes through the point $(-1, 4)$ and has the symmetric line $x = -2$. **Sketch** the graph of $f(x)$.

---

13. **Find** the values of a and b so that the graph of $f(x) = a(x-b)^2 - 4$ intersects the x-axis at 1 and 5. **Sketch** the graph of $f(x)$.

---

14. **Find** a quadratic function $f(x) = ax^2 + bx + c$ (or $f(x) = a(x-h)^2 + k$) for each parabola described below. **Draw** the parabola and **label** all information you have.

(1) min. value of -2 and x-intercepts of -1 and 3

(2) symmetric line is $x = 2$ and passes through two points $(4, 3)$ and $(3, 0)$

---

15. One number is four less than twice another number. Find the two numbers so that the sum of their squares is as small as possible.

16. A parking lot is to be formed by fencing in a rectangular plot of land except for an entrance 12 m wide along one of the sides. Find the dimensions of the lot of the greatest area if 600 m of fencing is to be used.

---

17. Use the remainder theorem and the synthetic division to find $P(-3)$ if $P(x) = 2x^4 - 4x^2 + 6x - 1$.

---

18. Use the factor theorem and the synthetic division to determine whether $x + 1$ is a factor of $P(x) = 2x^3 - 3x^2 - 8x - 3$.

---

19. Suppose that -1 is a zero of $P(x) = x^3 + 8x^2 + 19x + 12$. Find all other zeros.

---

20. Let Let $P(x) = (x + 6)(x^3 - 4x^2 + x - 4)$. Find all zeros of $P(x)$.

---

21. If the polynomial $P(x) = ax^4 + x^3 - x^2 - 2bx$ has the polynomial $x^2 - x - 2$ as a factor, find the values of a and b.

---

22. Solve the equation $2x^3 + x^2 + 5x + 12 = 0$

---

23. Given $P(x) = 2x^3 - 7x^2 - 27x - 18$,

    (1) list all possible rational zeros;
    (2) use synthetic division to test the possible rational zeros and find an actual zero;
    (3) use the quotient from part (b) to find the remaining zeros.

---

24. Show that $\sqrt{5}$ is an irrational number, using the Rational Zero Theorem.

---

25. Find a polynomial with the least degree that has real coefficients with indicated information. Express your answers in the form:

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

    (1) $P(2) = 3$
    (2) zeros: -2 (multiplicity 2) and $3i$

    (3) Sketch the graph by using the intercepts and end behavior. (Don't use a calculator!)